

# The Interpretation of Wave Mechanics with the help of Waves with Singular Regions\*

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## Abstract

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The homage paid to the great theoretical physicist that is Max Born highlights the role he has played in contemporary physics by introducing the probabilistic interpretation of the wave  $\psi$  of wave mechanics. In the article he has written for this felicitation volume, Einstein has summarized some of his objections to the adoption of the “purely probabilistic” interpretation of quantum mechanics which has developed out of the works of thinkers such as Born, Bohr and Heisenberg. I wish to recall briefly what my ideas on this question formerly were and why I have recently undertaken a fresh examination of these old ideas.

Between 1924, when my doctoral thesis was published, and 1927, I have tried to develop a causal and objective interpretation of wave mechanics by admitting the hypothesis of “double solution” according to which the linear equations of wave mechanics allow two kinds of solutions: the continuous solutions  $\psi$  usually considered, whose statistical nature was then beginning to emerge clearly thanks to the work of Born, and singular solutions which would have a concrete meaning and which would be the true physical representation of particles. The latter would be well localized in agreement with the classical picture, but would be incorporated in an extended wave phenomenon. For this reason, the particle’s movement would not follow the laws of classical mechanics according to which the particle is subject only to the action of forces which act on it along its trajectory, and does not suffer any repercussion from the existence of obstacles which may be situated far away outside its trajectory: In my present conception, on the contrary, the movement of the singularity should experience the influence of all the obstacles which hinder the propagation of the wave with which it is connected. This circumstance would explain the existence of the phenomena of interference and diffraction [2].

However, the development of the theory of the double solution presented great mathematical difficulties. For this reason, I contented myself with a simplified form of my ideas, to which I gave the name “pilot-wave theory”, and which coincided with the hydrodynamic interpretation of wave mechanics proposed at about the same time by Madelung [7]. I have presented this softened form of my ideas at the October 1927 Solvay Physics Conference. My presentation was the object of numerous criticisms notably on the part of Pauli. Pauli’s objections did not appear to me as being decisive, but soon thereafter I recognized that the pilot-wave theory was faced with a difficulty which seemed and still seems to me to be insurmountable.

The wave  $\psi$  used in wave mechanics cannot be a physical reality: its normalization is

arbitrary, its propagation is supposed to take place in general in a visibly fictive configuration space, and according to Born's ideas, it is only a representation of probability depending on the state of our knowledge and is suddenly modified by the information brought to us by every new measurement. Thus, with only the help of the pilot-wave theory, one cannot obtain a causal and objective interpretation of wave mechanics, by supposing that the particle is guided by the wave  $\psi$ . For this reason, since 1927 I had entirely come round to the purely probabilistic interpretation of Born, Bohr and Heisenberg.

A year and a half ago, David Bohm took up the pilot-wave theory again. His work is very interesting in many ways and contains an analysis of the measurement process which appears to be capable of answering an objection Pauli had raised to me in 1927 [4]. But since Bohm's theory regards the wave  $\psi$  as a physical reality, it seems to me to be unacceptable in its present form. Reiterating the arguments I have recalled above, Takabayasi, while demonstrating the interesting aspects of Bohm's ideas, has recently insisted on the impossibility of admitting the principle which is its point of departure [5]. But my first theory of the "double solution" does not seem to me to hurtle against the same difficulties, because it distinguishes the singular wave, which alone is endowed with physical reality, from the wave  $\psi$ , which in this theory, is but a statistical representation of the relative probabilities of the possible movements of these singularities. J.P. Vigi er having brought to my notice the analogies which existed between my considerations of 1927 and Einstein's ideas on the movement of particles considered as some kinds of field singularities in general relativity, I undertook once again the study of my old ideas about singular solutions. I will first recall the principles of my attempt of 1927. If  $\psi = Re^{iS/\hbar}$  is a solution of the wave equation of the usual continuous type, we also admit the existence of solutions of the form:

$$u(x, y, z, t) = f(x, y, z, t)e^{\frac{iS(x, y, z, t)}{\hbar}} \quad (1)$$

where  $S(x, y, z, t)$  is the *same* function as in  $\psi$  and where  $f$  represents an in general mobile singularity. The substitution of the solution  $u$  into the wave equation leads to the relation:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial n} \frac{1}{m} |\nabla S| + \frac{1}{2m} f \Delta S = 0 \quad (2)$$

at least in the non-relativistic case: the variable  $n$  is calculated along the normal to the

surface  $S = \text{constant}$ . It is natural to suppose that, if the singularity is approached along this normal,  $f$  and  $\frac{\partial f}{\partial n}$  are very large and that  $\frac{\partial f}{\partial n} \gg f$ . Then, for the velocity of the particle,  $\vec{v}$ , we find the fundamental formula

$$\vec{v} = \frac{\frac{\partial f}{\partial t}}{\frac{\partial f}{\partial n}} = -\frac{1}{m} \nabla S \quad (3)$$

Thus everything happens as *if* the particle were guided by a wave  $\psi$ , as in the pilot-wave theory revived by Bohm. However, here, inasmuch as  $S$  is also the phase of the wave  $u$  which has physical reality, the same objections do not arise.

Since the quantity  $|\psi|^2 = R^2$  obeys the well-known continuity equation

$$\frac{\partial R^2}{\partial t} + \nabla \cdot (R^2 \vec{v}) = 0 \quad (4)$$

where  $\vec{v}$  is given by equation 3. It is natural to suppose that  $R^2$  gives the probability of the presence of the singularity at a point when it is not known which of its trajectories is being described. Thus we come back to Born's hypothesis about the statistical meaning of  $|\psi|^2$ . This hypothesis appears here to be somewhat analogous to the one made in statistical mechanics, when only on the basis of Liouville's Theorem, one admits the equal probability of equal volumes of phase space. But a more complete justification appears to be necessary: in a recent memoir [6], Bohm has spelled out an argument which seems to lead to this justification.

We may add that my theory of the double solution leads us, as I have already shown in 1927, to consider the movement of the particle as taking place under the action of classical forces augmented by a quantum force derived from a potential:

$$U = -\frac{\hbar^2}{2m} \frac{\square R}{R} = -\frac{\hbar^2}{2m} \frac{\square f}{f} \quad (5)$$

where the equality of the two expressions for  $U$  follows from the hypothesis of the equality of the phases of  $\psi$  and  $u$ . The potential  $U$  is the "quantum potential" of my 1927 theory rediscovered by Bohm in his memoir.

It is evidently necessary to be able to extend the theory of the double solution to the case of the Dirac equations for electrons with spin. After a first attempt I made in this direction, Vigier made a second which now seems preferable to me [7]. In any case, it does not seem to me that the extension of the double solution to the Dirac equations raises essential difficulties.

However it is necessary to demonstrate the existence of solutions of the type  $u$ . Now an argument due principally to Sommerfeld shows that in general in a problem involving quantized states there do not exist singular solutions to the *linear* equations of wave mechanics having the same frequency as that of a stationary wave  $\psi$ . This result proves that it is not possible to consider the wave  $u$  as a solution of these linear equations possessing a singularity in the usual sense of the word, as I did in 1927. But it is possible to overcome this objection by using the term “singularity” to mean a very small singular region where the function  $u$  takes values so large that the equation it satisfies in this region is non-linear, with the linear equation valid everywhere for  $\psi$  being valid for  $u$  only outside the singular region [9]. This change in point of view does not alter the validity of the guiding formula (3), for which we can give a proof more rigorous than the one sketched above. Vigier thinks that one could thus reconcile the theory of the double solution with the ideas of Einstein, who has always tried to represent particles as singular regions of the field, and probably also with the non-linear electromagnetism of Born. Although it is not yet possible to pass definitive judgments on Vigier’s attempts, they allow us to entertain hopes of seeing the theory of General Relativity and that of Quanta united within the framework of a single representation in which causality will be reestablished.

An important point would be to justify the use of the formula (3) in the case of systems of interacting particles,  $S$  then being the phase of the wave  $\psi$  in configuration space  $\vec{v}$  the velocity of the representative point in this space. It would be necessary to show that this results from interactions between wave singularities of the type  $u$  evolving in three-dimensional physical space. In my article in the May 1927 issue of the Journal de Physique, I have sketched a proof of this kind, considering the configuration space as being defined by the coordinates of the singularities. In this way one is able to represent the movement of the interacting singularities as taking place in the physical space without necessarily making appeal to the configuration space. This fictitious space and the propagation of the wave  $\psi$  in it become necessary only for statistical expectation values. Following this line of argumentation, one should be able to obtain a physical interpretation of the Pauli principle if it could be shown that for fermions the wave  $u$  can involve only one singularity, whereas it could have several in the case of bosons. I have recently been able to elaborate some considerations which I think constitute a slight advancement in this direction [8].

The existence of singular regions of the wave  $u$  (whose dimensions most probably will

be of the order of  $10^{-13}$  cm) may permit us to endow elementary particles with a structure whose lack can be felt today in quantum theories, and probably even resolve difficulties pertaining to infinite energies. As Born has remarked, it is possible that in the atomic nucleus may be present conditions not foreseen by current theories: in our way of thinking, they will be due to an overlapping of the singular regions of the constituents of the nucleus. One can also foresee that the statistical role of the wave  $\psi$  will not be valid in the case of particles animated by movements so rapid that the length of their associated wave will attain dimension comparable to those of their singular regions.

Before I conclude, I would like to say a word on the subject of the objection raised by Einstein in his article against the formula (3), an objection that applies both to the theory of the double solution and to the pilot-wave theory. Like him, let us consider a wave  $\psi$  constrained to propagate along an axis  $Ox$  and to reflect off two perfectly reflecting mirrors placed perpendicularly to the axis at  $x = 0$  and  $x = l$ . The stationary forms of the wave  $\psi$  are given by:

$$\psi_n = a_n \sin \frac{n\pi x}{l} e^{\frac{iE_n t}{\hbar}} \quad (6)$$

When one of these stationary states is realized, equation (3) gives  $\vec{v} = 0$ . The particle associated with the stationary wave is immobile. Now, says Einstein, this consequence of equation (3) is inadmissible because it should be exact whatever the mass of the particle, and if this particle has a macroscopic mass and constitutes say a small ball, it is well known that its movement should consist of a to and fro motion along the  $Ox$  axis, with alternate rebounds from each mirror. Without going into a discussion of Einstein's very interesting argument in all its generality, I will limit myself to the following remark. In order for an expression such as (6) to be physically valid, it is necessary that the plane surface of the two mirrors be well defined at the scale of the wavelength. The mirrors are necessarily made of atoms in thermal movement, and as a consequence the precision with which the surfaces are defined cannot be greater than a fraction of an angström. The condition

$$\lambda = \frac{h}{mv} > 10^{-9}$$

in c.g.s. units shows that if the particle has macroscopic mass (say greater than  $10^{-9}$  grams), the velocity  $v$  should be practically zero. In order for the expression (6) to be considered as valid for a particle of macroscopic mass, its velocity must be practically zero, and the value given by (3) is practically satisfied.

In conclusion, I recognize that many difficulties still stand in the way of the adoption of the theory of the double solution. However, in spite of the risk of an ultimate failure, it does not seem to me to be useless to take up again the ideas I have recalled, to see whether, when suitably modified or completed, they can provide a causal and objective interpretation of wave mechanics, in accordance with the wish expressed many times by Einstein. If some day such a thing were to come to pass, it would of course hardly detract from the importance of the discovery made by Born the day he apprehended the statistical meaning of  $\psi$  in the usual wave mechanics.

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- [1] A. Einstein, this volume, p. 33.
  - [2] Louis de Broglie, *Compt. Rend.* **183**, 447 (1926). Louis de Broglie, *Compt. Rend.* **184**, 273 (1927). Louis de Broglie, *Compt. Rend.* **185**, 380 (1927). Louis de Broglie, *Journal de Physique*, series VI, t. 8, p. 225, (May 1927).
  - [3] Louis de Broglie, *Electrons et Photons*, Report to the Vth Solvay Conference, Gauthier Villars, p. 115 (1930). *Introduction à l'étude de la Mécanique ondulatoire*, Hermann, Paris, 1930 (English edition, Methuen, London).
  - [4] David Bohm, *Physical Review*, **85**, 166 and 180 (15th January 1952).
  - [5] Takehiko Takabayasi, *Progress of Theoretical Physics*, **8**, 143 (August 1952).
  - [6] David Bohm, *Physical Review*, **89**, 458 (15th January 1953).
  - [7] Louis de Broglie, *Compt. Rend.* **235**, 557 (1952). Jean-Pierre Vigier, *Compt. Rend.* **235**, 1107 (1952).
  - [8] Louis de Broglie, *Compt. Rend.* **235**, 1345 (1952); **235**, 1453 (1952).
  - [9] Louis de Broglie, *Compt. Rend.* **236**, 1453 (1952).